Introduction

When viewed from distance, large parts of the topography of landmasses and the bathymetry of the sea and ocean floor can be regarded as a smooth background with local features. Locally Refined B-splines (LR B-splines) is a novel format that is able to provide a compact and accurate representation of smooth shapes whilst also providing extra modelling freedom in areas where more flexibility is required. In this work, we have explored:

- Methods for approximation of geographical point clouds by LR B-splines, which can drastically reduce the data size whilst maintaining control over the approximation error;
- Tiling of the LR B-spline surfaces to enable scalability to truly ‘big’ datasets;
- Initial thoughts as to how hybrid representations including LR B-spline, triangulations and point clouds can be used as a compact representation of smooth underlying terrain, whilst maintaining sharp features such as buildings and other complex structures;
- The potential for automating the creation of a hybrid surface representation from raw point data by classifying regions according to their distance from the LR B-spline surface.

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Adaptive surface approximation

The point cloud is initially approximated with a lean LR B-spline surface on a tensor-product domain. An iterative algorithm is used to refine and re-approximate the surface, until a given tolerance is met or a specified number of iterations is performed.

At each iteration level (see Figures to the right), either of two approximation methods can be employed:

1. Least squares approximation with a smoothing term: (this method is global, so it is not suited for very large LR B-splines, but provides better approximations).
2. Multilevel B-spline approximation adapted for LR B-splines (this method is local and is stable to high levels of iteration).

The refinement strategy at each approximation level determines how the domain is split in order to increase the number of degrees of freedom in regions that are not within a given tolerance. The theory of LR B-splines allows for a huge amount of flexibility in the choice of refinement technique. This contrasts with the tensor-product approach, which requires a global refinement over the entire parameter domain in either direction.

Local refinement of splines

There are three main approaches to the local refinement of splines:

1. (Truncated) hierarchical B-splines (Forsey et al., 1988, Giannelli et al., 2012)
2. T-splines (Sederberg et al., 2003)
3. Locally Refined B-splines (LR B-splines) (Dokken et al., 2013)

In this work we follow the latter method of LR B-splines. An LR-B spline surface $F$ is expressed with respect to parameters $u$ and $v$ as

$$F(u,v) = \sum_{i,j} N_{i}^{r}(u) N_{j}^{s}(v) P_{i,j},$$

where $P_{i,j}$ are the surface coefficients, $s_j$ are scaling factors to enable partition of unity and $N_{i}^{r}(u) N_{j}^{s}(v)$ are the associated B-splines with degrees $d_i$ and $d_j$ in the respective parameter directions. The B-splines are defined on knots which represent the joints between the polynomial patches. In contrast to tensor product spline surfaces, the knots lines are local. This results in a parameter domain composed by a number of boxes.

Example: Liguria

Data courtesy Regione Liguria and CNR-IMATI

- Topographic data from Liguria, Italy.
- The data set contains 3,045,671 points.
- The region covered by the data is dominated by mountainous terrain and sharp features such as buildings.

We classify the points according to their distance from the surface approximation (tolerance of 0.5 m, six levels of iteration). This intermediate level of refinement provides enough detail to show the trends of the underlying terrain, without picking out the finer details. The classification of points can be used to indicate the most suitable type of representation. The mainly white areas show the smooth background (LR B-splines), whilst the areas with red/green indicate buildings (triangulations) or vegetation (scattered red points). Deep red points lie well above the ground and should be modelled separately.

Example: Bathymetry data set

Data courtesy HR Wallingford: SeaZone

- Bathymetric dataset containing 58,578,420 points
- Covers a region of approximately 8 km x 12 km
- Variation in z-direction is approximately 30 m

This example shows that the data size of the LR B-spline approximation is relatively stable regardless of the density of the input point set.

We successively thin the point cloud and approximate with an LR B-spline surface of the same tolerance (0.5 m) and the same number of iteration levels (six levels).

It can be seen that the resulting numbers are fairly constant. The proportion of out-of-tolerance (OOT) points reduces very slightly with thinning until the point cloud is thinned to 1/16th of its original size. At that point, significant details begin to be lost.

<table>
<thead>
<tr>
<th>Number of points</th>
<th>Number of polynomial patches</th>
<th>Maximum error (all points)</th>
<th>Average error (OOT points)</th>
<th>Proportion of OOT points</th>
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</thead>
<tbody>
<tr>
<td>58 578 420</td>
<td>59 458</td>
<td>5.55 m.</td>
<td>0.092 m.</td>
<td>0.66 m.</td>
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<tr>
<td>29 289 210</td>
<td>58 993</td>
<td>5.39 m.</td>
<td>0.092 m.</td>
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<td>14 644 604</td>
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<tr>
<td>3 661 551</td>
<td>60 667</td>
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Local refinement allows maintaining smoothness and details in the same surface. Each knot is associated with a set of coefficients, which are used to model the surface. The 3D model can be refined in a number of iterations.

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